Closing Tue: TN 4
Closing next Thu: TN 5 (Last HW)
Final: Sat, June 3rd, 5:00-7:50pm, KANE 130

TN 5: Using Taylor Series
Idea: Manipulate the 6 series we now know to get other series.

Tools:
1. Substitute (replace x by something else)
2. Integrate; note that
$$\int x^n \, dx = \frac{1}{n+1} x^{n+1} + C$$
3. Differentiate; note that
$$\frac{d}{dx}(x^n) = nx^{n-1} + C$$
4. Combine; note that
$$\sum_{k=0}^{\infty} kx^k - 3\sum_{k=0}^{\infty} \frac{1}{k!} x^k = \sum_{k=0}^{\infty} \left(k - \frac{3}{k!}\right) x^k$$

Here are the 6 series you can quote:
For the first three below **open interval of convergence: -∞ < x < ∞**

$$e^x = \sum_{k=0}^{\infty} \frac{1}{k!} x^k$$
$$\sin(x) = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!} x^{2k+1}$$
$$\cos(x) = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!} x^{2k}$$

For these three below **open interval of convergence**: **-1 < x < 1**

$$\frac{1}{1-x} = \sum_{k=0}^{\infty} x^k$$
$$-\ln(1-x) = \sum_{k=0}^{\infty} \frac{1}{k+1} x^{k+1},$$
$$\arctan(x) = \sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1} x^{2k+1}$$

Substitution Questions: For each below: find the Taylor series based at b = 0, first three nonzero terms and give the interval of convergence:

(a) $f(x) = 3e^{2x}$

(b) $g(x) = \frac{5}{1-4x}$

(c) $h(x) = \frac{3}{2x+1}$

Combining and Working with sums

For each below:

find the Taylor series based at b = 0, first three nonzero terms and give the interval of convergence:

(a) $y = 7 + 3x^5 e^{2x}$

(b) $y = \frac{5}{1-4x} - \frac{3}{2x+1}$

(c) $y = \cos^2(x)$ (Big hint: Half-angle)

Integrating

(a) Give the first three nonzero terms of the Taylor Series for

$$\int_0^x 7 + 3t^5 e^{2t} dt$$

(b) Find a Taylor series for (from HW):

$$A(x) = \int_{0}^{x} \frac{\sin(t)}{t}\, dt$$

Steps:

1. What is A'(x)?
2. Give the Taylor series for sin(t) at b=0.
3. What can you say about sin(t)/t?
4. Integate.